Transistor

a TFHE-friendly stream cipher

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29th October 2024

Outline

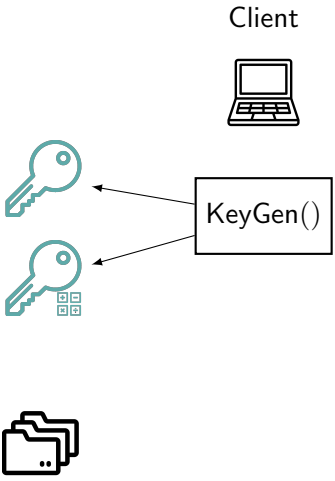
- FHE and the bandwith problem
- Transciphering
- TFHE and a wishlist
- Design of `Transistor`

FHE in one slide

Client

Server

FHE in one slide



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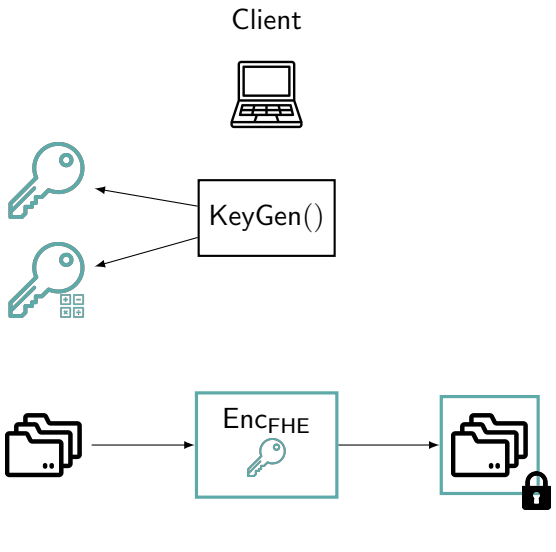

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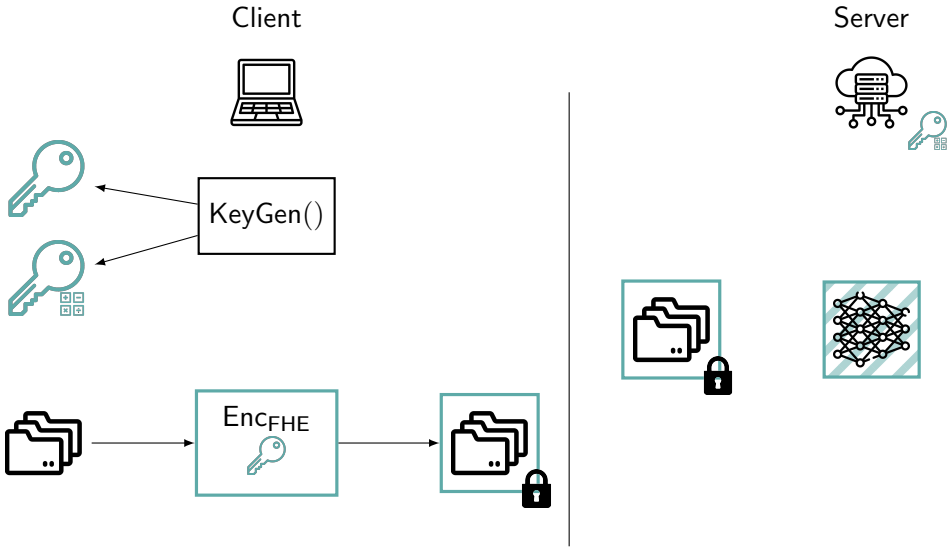

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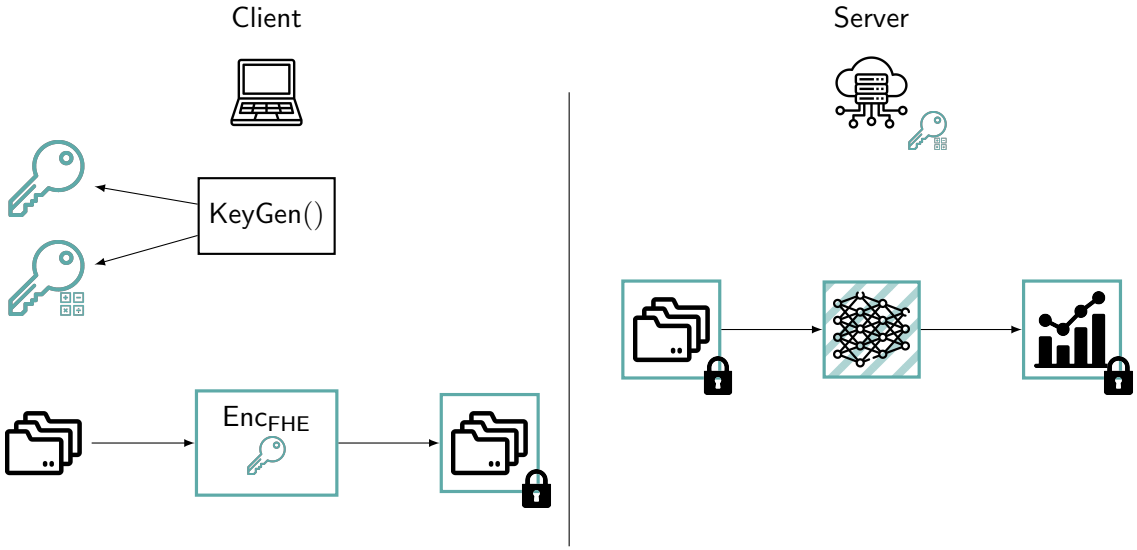

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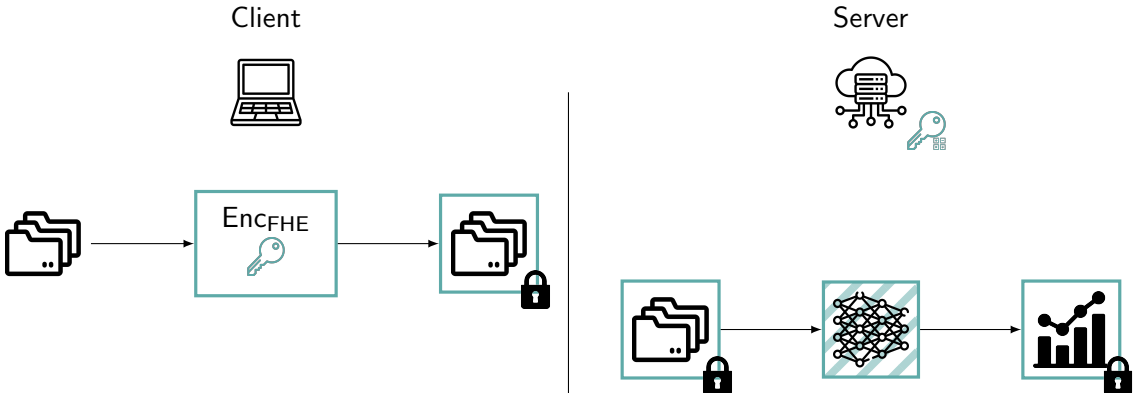

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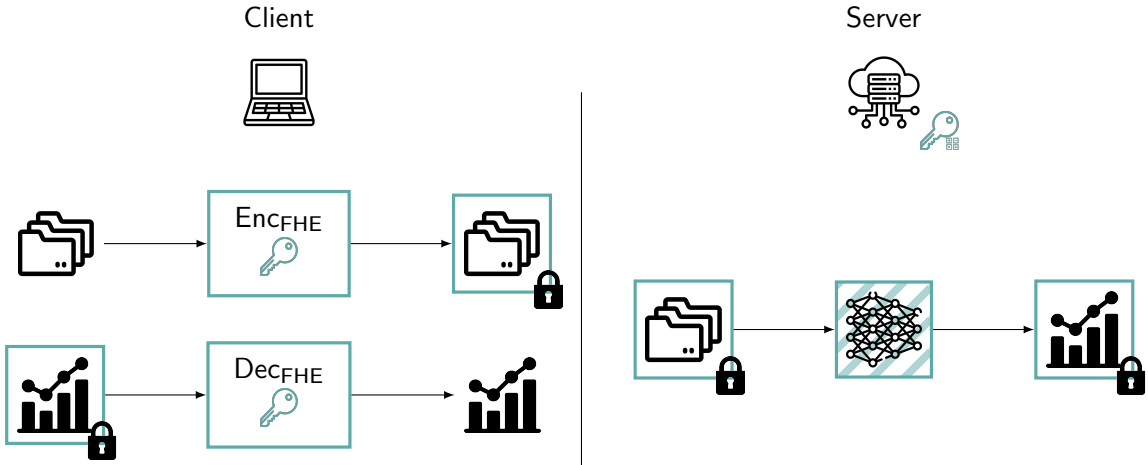

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How to reduce this bandwith overhead ?

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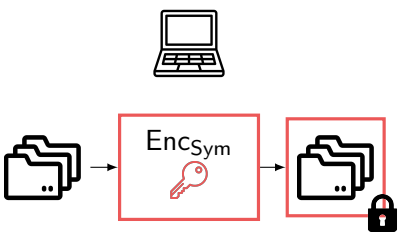

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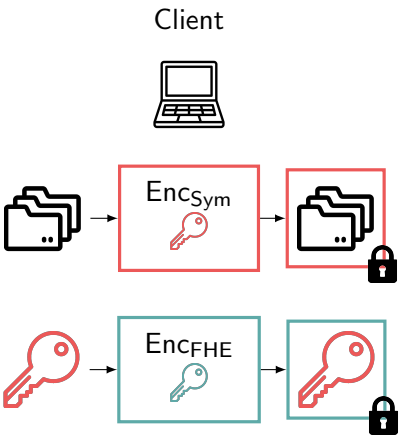

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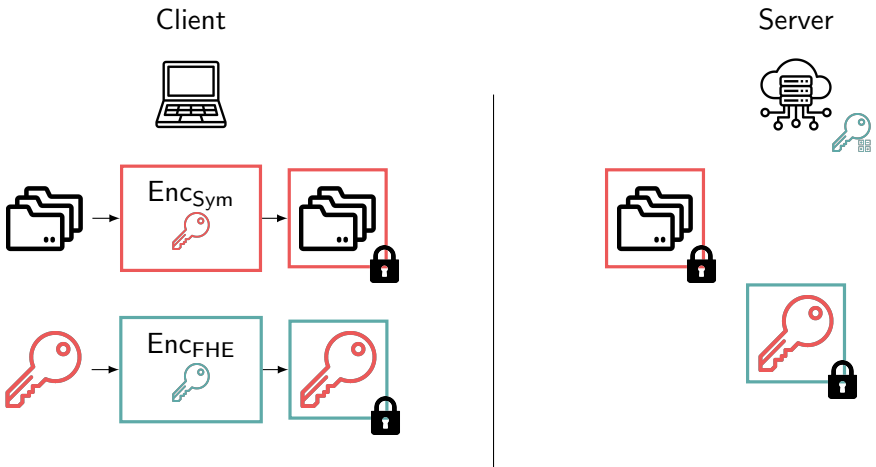

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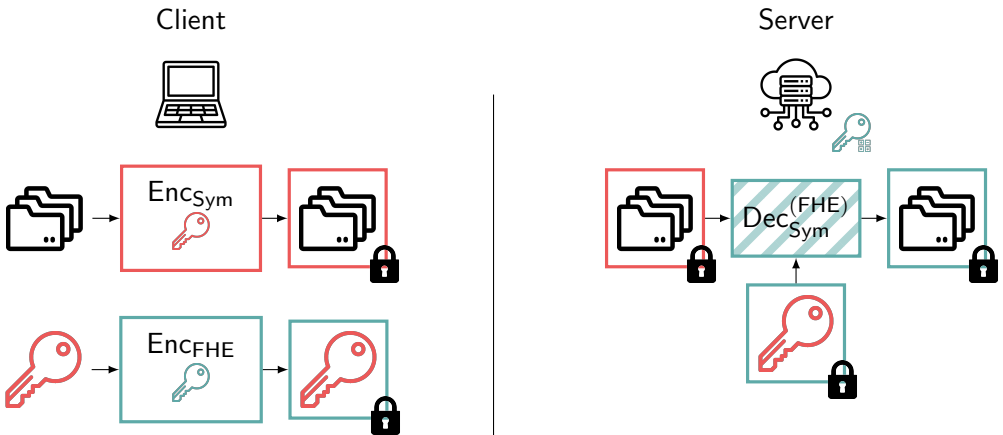

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With `Transistor`, we look for the best of both worlds (fast in FHE and secure)

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- **Negacyclicity problem:**
  - If p is **even**, restricts some homomorphic operations.
  - Disappears when p is **odd**

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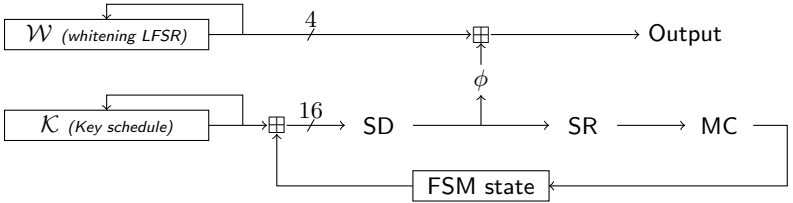

(a) General structure (rectangles correspond to registers).

(b) SD.

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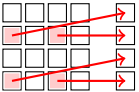

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Figure: A high level view of `Transistor`.

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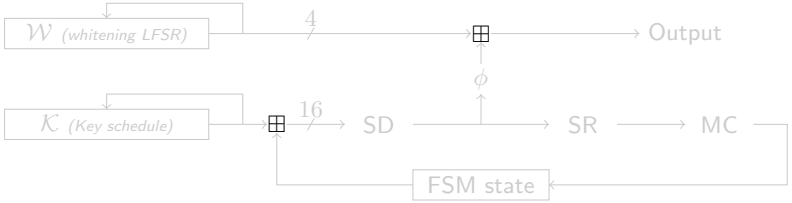

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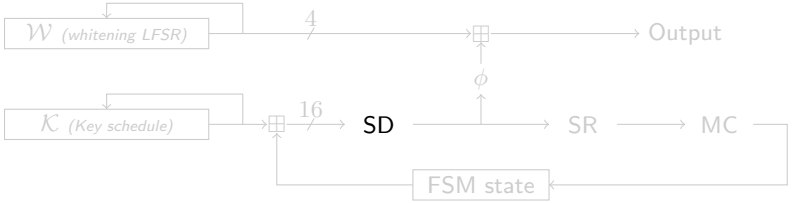

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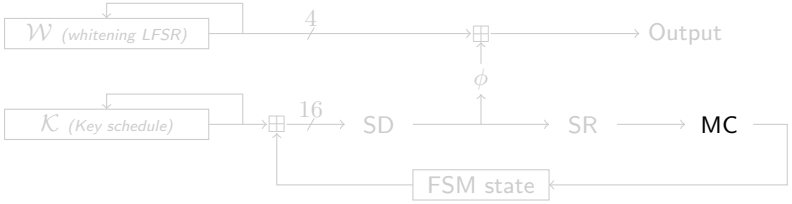

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MixColumns

We opted for a 4×4 Maximum Distance Separable (MDS) to ensure optimal diffusion. The matrix we chose is

$$M = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -1 & 1 & -2 \\ 1 & 1 & -2 & -1 \\ 1 & -2 & -1 & 1 \end{bmatrix} . \tag{1}$$

We verified that there is no MDS matrix in $\mathbb{F}_{17}$ with coefficients in $\{-1, 1\}$ by exhaustively testing all such matrices. By testing all matrices with coefficients in $\{-2, -1, 1, 2\}$, we found a total of $30\,720$ MDS matrices with an ℓ_2-norm of 7. We selected M for its symmetries, particularly because it is its own transpose.

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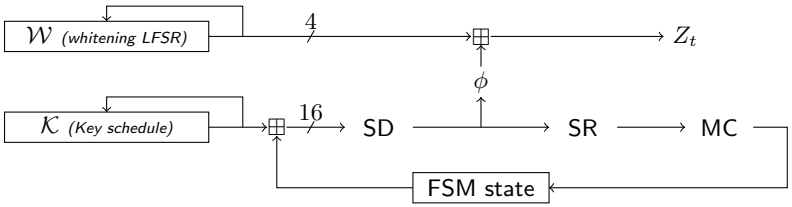

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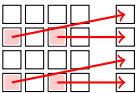

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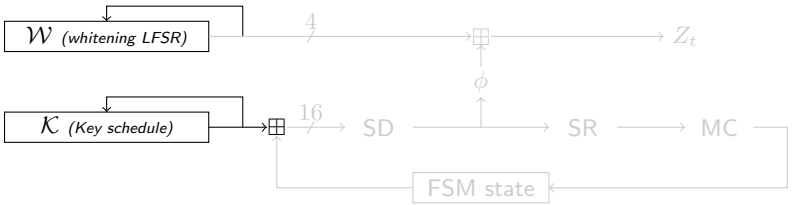

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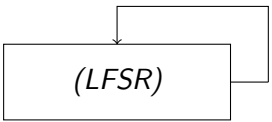
Silent LFSR



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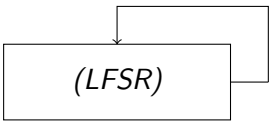

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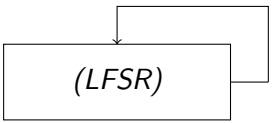

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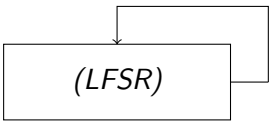

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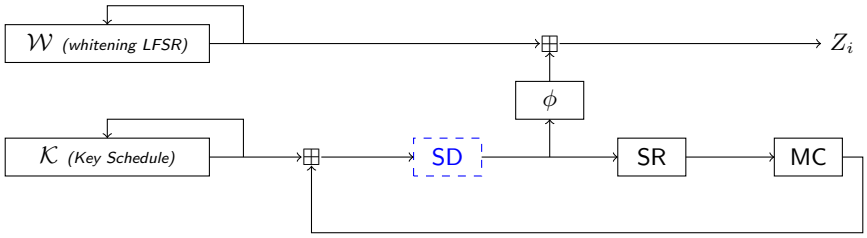
The noise variance in the output of the silent LFSR remains stable over time, without using any PBS.

Noise Management



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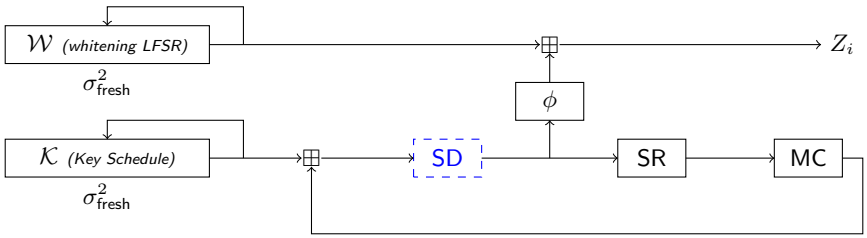

The LFSR are loaded with fresh ciphertexts of noise σ_{fresh}

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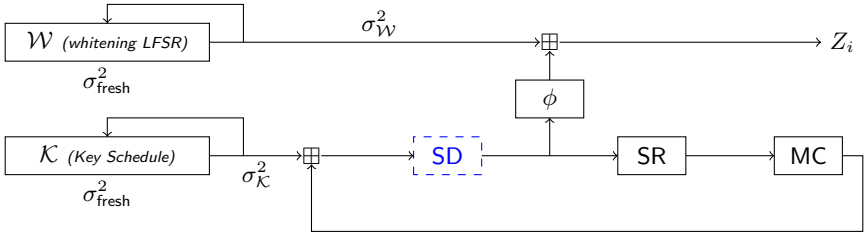

The output of *silent* LFSRs have noise

$$\sigma^2_{\mathcal{K}} \leq |\mathcal{K}| \cdot \left(\frac{p-1}{2}\right)^2 \cdot \sigma^2_{\text{fresh}} \quad \text{and} \quad \sigma^2_{\mathcal{W}} \leq |\mathcal{W}| \cdot \left(\frac{p-1}{2}\right)^2 \cdot \sigma^2_{\text{fresh}}$$

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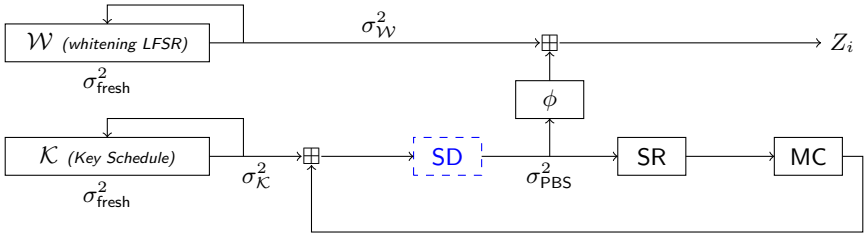

The output of a PBS has noise **independent** from the noise of the input. We just have:

$$\sigma_{\text{PBS}} \gg \sigma_{\text{fresh}}$$

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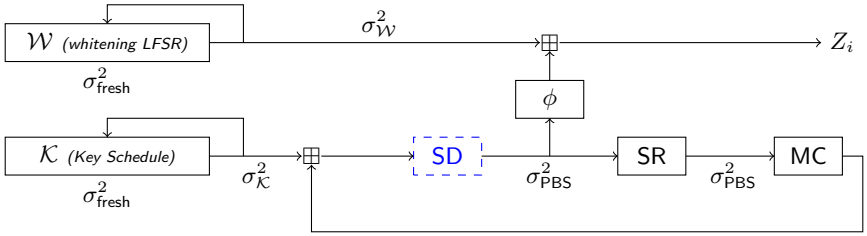

ShiftRows simply reorders the ciphertexts, thus adds no noise.

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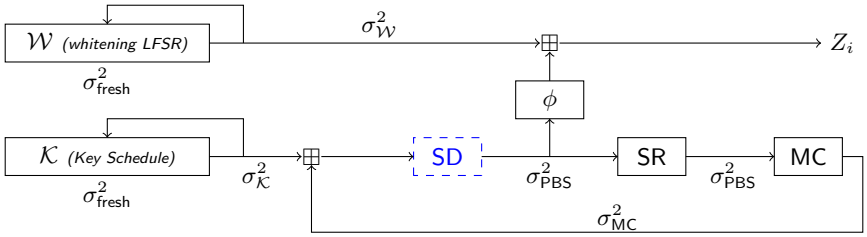

The variance is multiplied by the norm of the diffusion matrix

$$\sigma^2_{\text{MC}} = L^2_{\text{MC}} \cdot \sigma^2_{\text{PBS}}.$$

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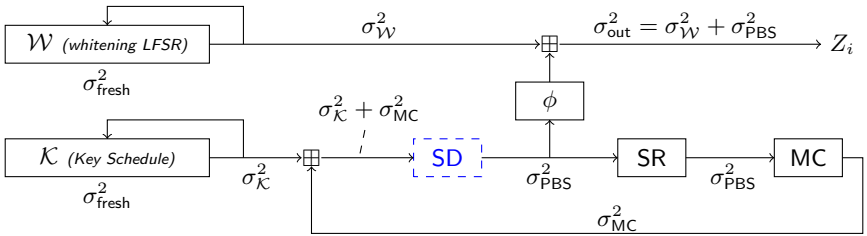
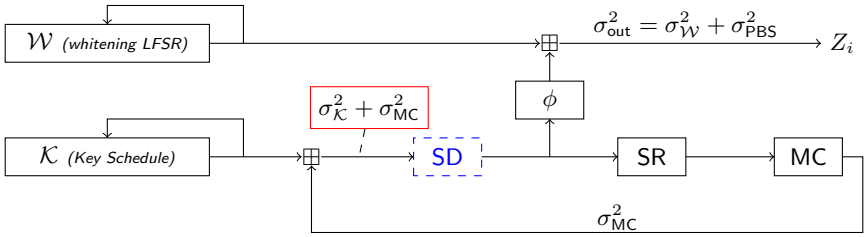


Variances are summed.

Noise Management



There are two "hot spots" where things can go wrong:

- Right before the PBS (risk of false bootstrapping result)

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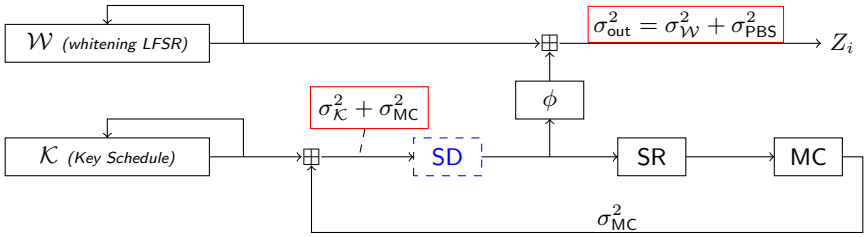

There are two "hot spots" where things can go wrong:

- Right before the PBS (risk of false bootstrapping result)
- At the output of the cipher (risk of producing a keystream of noise too high, thus unusable)

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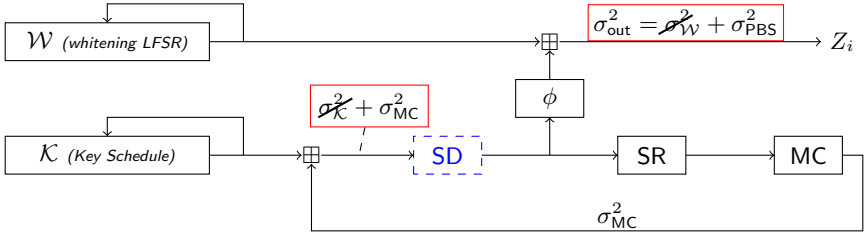

As $\sigma_{\text{MC}} \gg \sigma^2_{\mathcal{K}}$ and $\sigma_{\text{PBS}} \gg \sigma^2_{\mathcal{W}}$, the noise produced by the LFSRs is negligible.

Takeaway 1: No Restriction on the size of the LFSRs

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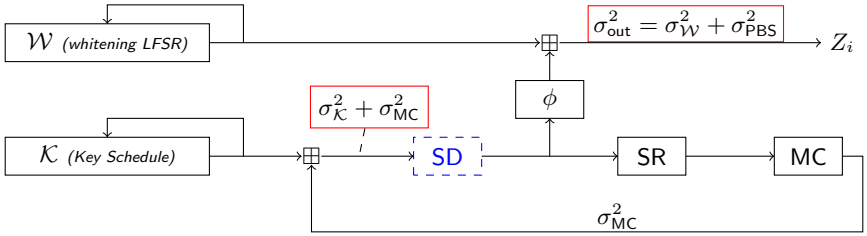

Takeaway 2 : Dimensioning the TFHE parameters for `Transistor` can be reduced to select parameters for a simple PBS

Performances

Table: Execution timings of `FRAST` and `Transistor`.

Cipher	p_{err}	Setup	Latency	Throughput
`FRAST`	2^{-80}	25 s (8 threads)	6.2 s	20.66 bits/s
`Transistor`	2^{-128}	No	251 ms	65.10 bits/s

Bonus: Takeways on cryptanalysis (Disclaimer: I have no clue)

A quick tour of the security arguments:

- Time-Memory-Data Trade-Offs put a limit on the length of the keystream we can generate with a single key: 2^{31}.
- Guess-and-Determine: as one quarter of the state is used to generate the keystream, one has to guess $\frac{3}{4}$ of the LFSR, which puts a lower bound on its size. $|\mathcal{K}| = 64$ and $\mathcal{W}| = 32$
- The paper demonstrates that **Three consecutive outputs are statistically independent of the secret key**. The proof only requires the matrix of MixColumns to be MDS instead of automatic search methods like in `Rocca`.
- Linear approximation, Fast Correlation attacks, Algebraic Attacks